[0001] Introduction to Simultaneous Nutation and Periodic Phase Encoding of Moving Spins ("SNAPPEMS")

[0002] Transient response of spin systems have usually been described by solutions to the Bloch equations (Slichter, C.P. "Principles of Magnetic Resonance", Third Edition. Springer-Verlog 1989, p.33, Ch. 2) which describes the magnetization as a function of both the applied magnetic fields and the relaxation effects on a phenomenological basis. Solutions to the Bloch equations require assumptions about the magnitude of these parameters and prescription of boundary conditions, creating a differential system applicable to a particular set of circumstances. To simplify this process further, the description below begins with the equation of motion of an isolated spin, modeled as gyroscopic precession creating a magnetic moment which interacts with moments created by applied magnetic fields, then introduces the total magnetization as a function of spin density, and finally limits the applicability of the equations so derived by the relationship between the times of the event sequence to the relaxation times.

[0003] Gyroscopic Precession

[0004] "A rigid body free in space without any constraints can rotate permanently only about a principal axis of inertia" (Den Hartog, J.P., "Mechanics". McGraw Hill Book Co., 1948. Dover Publications Inc. 1961, p.315 No XII). "If a rigid body rotates with speed ω_1 about a principal axis of inertia, and with $\omega_2 = \omega_3 = 0$ about the other two principal axes, then the angular momentum vector (i.e. moment of momentum vector) $\vec{M}$ has the same direction as the angular-speed vector $\vec{\omega}_1$ (which is along the axis of rotation)," (Hartog, J.P., op. cit., p.315 No VIII). "Angular velocities of a rigid body about various axes in space, all intersecting in a point, can be compounded vectorially into a resultant angular speed about THE axis of rotation" (Hartog, J.P., op. cit. p.314 No III).

[0005] We infer from the last statement that the resultant angular speed ω_1 about THE axis of rotation taken as the resultant angular velocity $\vec{\omega}$ can be decomposed into a vector sum of angular velocities $\sum \overrightarrow{\omega_n}$. Further, taking the rigid body as having mass symmetrically distributed about all axes through the center of mass yields a constant scalar moment of inertia I_0 about all axes, leading to the desired result decomposing the angular momentum vector $\vec{M}$:

$$\vec{M} = I_0\vec{\omega} = \sum_n I_0\overrightarrow{\omega_n}$$

[0006] From Newton's equations (Hartog, J.P., op. cit., p.277, eq.27b), with $\overrightarrow{M_G}$ being the moment of external forces about the center of mass:

$$\overrightarrow{M_G} = \frac{d}{dt} I_0\vec{\omega} = \sum_n \frac{d}{dt} I_0\overrightarrow{\omega_n} .$$

[0007] Thus the vector sum of the moments of a set of external forces $\sum \overrightarrow{M_{G_n}}$ equals the time rate of change of the total angular momentum $I_0\vec{\omega}$, i.e:

$$\vec{M}_G = \sum_n \overrightarrow{M_{Gn}} = \frac{d}{dt} I_0\vec{\omega} = \sum_n \frac{d}{dt} I_0\overrightarrow{\omega_n} .$$

[0008] Gyromagnetic Ratio

[0009] Taking the rigid body as having a symmetrical distribution of charge about the center of mass creates a magnetic moment $\vec{\mu}$ about any axis of rotation proportional to the angular velocity $\vec{\omega}$ about that axis of rotation (Smythe, W.R. "Static and Dynamic Electricity", Second Edition. McGraw Hill Book Co 1950, p.260) where each element of charge dq at distance r from the axis of rotation creates an element $d\vec{\mu}$ of this magnetic moment $\vec{\mu}$ where, by definition of the magnetic moment,

$$d\vec{\mu} = (\pi r^2)(\ r\vec{\omega})\frac{dq}{2\pi r}$$

(Balanis, C.A., "Advanced Engineering Electromagnetics". John Wiley and sons 1989, p.87, eq.2-82).

[0010] Each element of mass dw at distance r from this axis of rotation creates an element of the angular momentum (moment of momentum) dM of (Hartog, op. cit., p.276)

$$d\vec{M} = (r)(\ r\vec{\omega})dw$$ (Balanis, op. cit., p.87, eq. 2-83).

[0011] The ratio is assumed to be a constant:

$$\frac{d\vec{\mu}}{d\vec{M}} = \frac{1}{2}\left(\frac{dq}{dw}\right) \equiv \gamma \qquad \text{(Balanis, op. cit., p.87, eq. 2-84).}$$

γ being a scalar constant termed the gyromagnetic ratio. Integrating, with boundary condition $\vec{\mu} = 0$ when $\vec{M} = 0$ yields:

$$\vec{\mu} = \ \gamma\vec{M}.$$

[0012] General Equation of Motion of an Isolated Spin

[0013] A magnetic moment $\vec{\mu}$ subjected to a field of magnetic induction $\vec{B}$ will experience a mechanical moment (torque) $\overrightarrow{M_G}$ such that:

$$\overrightarrow{M_G} = \vec{\mu}\times\vec{B} \qquad \text{(Smythe, W.R., op. cit., p.261)}$$

[0014] In free space of permeability μ_0, the magnetic induction $\vec{B}$ is proportional to the magnetic field intensity $\vec{H}$:

$$\vec{B} = \mu_0\vec{H}$$

[0015] Equating the rate of change of the angular momentum to the applied mechanical moment (torque) yields

$$\vec{\mu}\times\vec{B} = \frac{d}{dt}\vec{M} \text{ (Slichter, op. cit., p.11, eq. 2.1).}$$

[0016] Multiplying by the gyromagnetic ratio γ and substituting $\vec{\mu} = \gamma\vec{M}$ and $\vec{B} = \mu_0\vec{H}$ yields

$$\vec{\mu}\times(\gamma\mu_0)\vec{H} = \frac{d}{dt}\vec{\mu}.$$

[0017] Defining $\gamma' = \ \gamma\mu_0$ yields the equation of motion of a magnetic moment (spin) subjected to a magnetic field intensity $\vec{H}$:

$$\vec{\mu}\times\gamma'\vec{H} = \frac{d}{dt}\vec{\mu}.$$

[0018] Dividing by μ yields the instantaneous angular velocity of a magnetic moment (spin) $\vec{\mu}$ subjected to a time varying ambient magnetic field intensity $\vec{H}$:

$$\vec{I}_\mu \times \gamma' \vec{H} = \frac{d}{dt} \vec{I}_\mu ,$$

which is a linear differential equation with constant coefficients, allowing superposition.

[0019] Decomposing $\vec{H}$ into H_0, h_p, and h_n

[0020] Define an orthogonal coordinate system for $\vec{I}_\mu$ and $\vec{H}$ as $\vec{z}, \vec{x}, j\vec{y}$ where $\vec{x} + j\vec{y}$ forms Gaussian planes everywhere orthogonal to $\vec{z}$. This allows decomposition of the ambient magnetic field intensity $\vec{H}$ such that:

$$\vec{H} = \left[H_0 + h_p \cos(\Omega \mathrm{t})\right]\vec{z} + \overrightarrow{h_n} e^{j\gamma' H_0 t} ,$$

where H_0 is a strong non-time variant ambient magnetic field intensity, h_p is co-aligned with H_0 and sinusoidally periodic at frequency Ω , and h_n is everywheres orthogonal to H_0 rotating in the local orthogonal $\vec{x}, j\vec{y}$ Gaussian plane at angular velocity $\gamma \mu_0 H_0 = \gamma B_0 = \omega_0$, which is the mean Larmor frequency.

[0021] Equation of Motion of SNAPPEMS

[0022] Substituting and rearranging in differential form

$$d\vec{I}_\mu = (\vec{I}_\mu \times \gamma' H_0 \vec{z})dt + (\vec{I}_\mu \times \left[\gamma' h_p \cos(\Omega t)\right]\vec{z})dt + \left(\vec{I}_\mu \times \gamma' \overrightarrow{h_n} e^{j\gamma' H_0 t}\right) dt$$

since the vector product (cross product) distributes across a vector sum.

[0023] The first term creates a constant precession of $\vec{I}_\mu$ of angular velocity $\gamma' H_0$ about the $\vec{z}$ axis.

[0024] The second term creates a periodic precession of $\vec{I}_\mu$ of peak angular velocity $\gamma' h_p$ and temporal frequency Ω about the $\vec{z}$ axis.

[0025] The third term represents a periodic precession of $\vec{I}_\mu$ of peak angular velocity $\gamma' h_n$ about an axis perpendicular to the $\vec{z}$ axis, said axis rotating with angular velocity $\gamma' H_0$ in the $\vec{x} + j\vec{y}$ Gaussian plane, a periodic Bloch-Siegert effect (Sacolick et. Al., Magn. Reson. Med. 2010 May; 63 (5): 1315-1322)

[0026] These three instantaneous angular velocities add vectorially to a resultant angular velocity, which when integrated over time, creates the locus of $\vec{I}_\mu$ in space.

If $\gamma' h_n \ll \gamma' h_p \ll \gamma' H_0$, the locus of the unit vector $\vec{I}_\mu$ describes a serpiginous line of colatitude θ and longitude φ on a unit diameter sphere, said sphere rotating with an angular velocity $\gamma' H_0$ in the local $\vec{x} + j\vec{y}$ Gaussian plane (Slichter, op. cit., Ch. 2.4, p.20). The rate of increase of θ is maximum at the poles (θ = 0, π) and decreases at mid-latitude to .340 of maximum at the equator (θ = π/2). The governing equations are:

$$\vec{1}_u \, X \, \gamma' \overrightarrow{h_n'} \triangleq \vec{\omega} = \frac{d\vec{\theta}}{dt} = \vec{1}_u X [\cos^2 \theta + \sin^2 \theta \cos^2 \varphi]^{\frac{1}{2}} \gamma' \overrightarrow{h_n}$$

$$\vec{1}_u X \gamma' \overrightarrow{h_p} = \vec{\omega}_p = \frac{d\vec{\varphi}}{dt} = \gamma' \overrightarrow{h_p} \cos \Omega t$$

$$\vec{\varphi} = \int \vec{\omega}_p \; dt = \frac{\gamma' h_p}{\Omega} \sin \Omega t; \frac{\gamma' \; h_p}{\Omega} \cong 1.8, \text{ to maximize } J_1 \text{ (vide infra).}$$

$$h_n' \triangleq h_n \left[\cos^2 \theta + (\sin^2 \theta) \left[\cos^2 \left(\frac{\gamma' h_p}{\Omega} \sin \Omega t \right) \right] \right]^{\frac{1}{2}}$$

$$As \;\; \theta \to (0, \pi) \; h_n' \;\to h_n$$

$$\gamma' h_n' \triangleq \gamma' h_n \left[1 - \sin^2 \theta \left[1 - \cos^2 \left(\frac{\gamma' h_p}{\Omega} \sin \Omega t \right) \right] \right]^{\frac{1}{2}}.$$

$$\frac{d\theta}{dt} = \gamma' h_n \left[1 - \sin^2 \theta \sin^2 \left(\frac{\gamma h_p}{\Omega} \sin \Omega t \right) \right]$$

$$\int_0^\tau \gamma' h_n dt = \int_0^\theta \left[1 - \sin^2 \theta \sin^2 \left(\frac{\gamma' h_p}{\Omega} \sin \Omega t \right) \right]^{-\frac{1}{2}} d\theta$$

$$\gamma' h_n \tau = \int_0^\theta [1 - \sin^2 \theta \sin^2 \varphi]^{-\frac{1}{2}} d\theta$$

$$\gamma' h_n \tau \triangleq F(\theta \backslash \varphi)$$

Which is an incomplete elliptic integral of the first kind with a periodic modular angle, and is not integrable. (Abramowitch, M., Stegun, 1. A. eds. “Handbook of Mathematical Functions” Ninth Edition, Dover Publications Inc. 1965, P589 eq. 17.2.6.)

[0027] Output Voltage in Receiver Coil

[0028] Define the transverse magnetization $\vec{\mu}_T$ as the projection of the magnetic moment $\vec{\mu}$ on the $\vec{x} + j\vec{y}$ Gaussian plane, which plane is transverse to the $\vec{z}$ axis, such that:

$$\vec{\mu}_T = (\vec{\mu}\sin\theta)e^{j\emptyset},$$

where θ is the colatitude of $\vec{\mu}$ with respect to the $\vec{z}$ axis, and φ is the longitude taken from a zero meridian through the $\vec{z}$ and $j\vec{y}$ axes. The instantaneous angular velocity of $\vec{u}_T$ in the $\vec{x} + j\vec{y}$ Gaussian plane then is:

$$\vec{\omega}_T = (\gamma' \hbar_p \cos(\Omega t) + \gamma' H_0)\vec{z}$$

creating a phase incrementation φ of $\vec{\mu}_{\mathrm{T}}$ at time t of

$$\vec{\varphi} = \int_0^\tau \vec{\omega}_T dt = (\frac{\gamma' \hbar_p}{\Omega} \sin(\Omega t) + \gamma' H_0 t\ )\vec{z}\,.$$

[0029] A coil of N turns will subtend the rotating magnetization of $\overrightarrow{\mu_T}$ such that:

$$\mu_c = \mu\sin\theta\,(\sin\varphi) = (\mu\sin\theta)\sin\left[\frac{\gamma' \hbar_p}{\Omega}\sin(\Omega t) + \gamma' H_0\ t\right].$$

[0030] By Faraday’s law, the voltage induced in the coil is

$$V = N\mu_0\frac{d\mu_c}{dt} = \quad N\mu_0\left((\mu\sin\theta)(\gamma' \hbar_p\cos(\Omega t) + \gamma'\ H_0)\left(\cos\left[\frac{\gamma' \hbar_p}{\Omega}\sin(\Omega t) + \gamma' H_0 t\right]\right)\right.$$
$$\left. + sin\left[\frac{\gamma' \hbar_p}{\Omega} sin(\Omega t + \gamma' H_0 t)\right](\mu\cos\theta)\left(\frac{d\theta}{dt}\right)\right)$$

Since $\dfrac{d\theta}{dt} \leq \gamma' \hbar_n \ll \gamma' \hbar_p \ll \gamma' H_0$:

$$V \cong N(\mu_0\mu\sin\theta)(\ \gamma' H_0)\left(\cos\left[\tfrac{\gamma' \hbar_p}{\Omega}\sin(\Omega t) + \gamma' H_0 t\right]\right),$$

where N is the number of turns in the coil, μ_0 is the permeability of free space, H_0 is the main magnetic field intensity, γ' is the gyromagnetic ratio $\mu_0\gamma$, h_p is the peak magnetic field intensity of the phase modulating field of temporal frequency Ω, and θ is the colatitude of the magnetic moment (spin) of magnetic field intensity μ.

[0031] The Fourier transform of V with respect to time is

$$\Im V|_\omega = \pi A \sum_{=-\infty}^{+\infty} \left[J_n\left(\frac{\gamma' h_p}{\Omega}\right) \delta_{[\omega-(\gamma' H_0+n\Omega)]} + J_n\left(\frac{\gamma' h_p}{\Omega}\right) \delta_{[\omega+(\gamma' H_0+n\Omega)]} \right],$$

where $A = N(\mu_0\mu)(\mu_0 H_0)\gamma \sin\theta$. (Poularikas, A.D. ed. "The Transforms and Application Handbook" CRC IEEE Press 1995, op. cit. p. 221, eq. 2.82)

[0032] Phase Modulating Field

[0033] Three voltages are induced in the receiver coil; the first by h_p at a low frequency Ω, the second by h_n of radio frequency (RF) frequency $\gamma' H_0 = \gamma\mu_0 H_0 = \gamma B_0 = \omega_0$, and the third by the precession of the magnetic moment $\vec{\mu}$ (spin) consisting of a central frequency ω_0 with an infinite number of sidebands spaced about this central RF Larmor frequency ω_0 at frequency intervals Ω. These sidebands permit adjustment of h_n for the maximum output voltage in the receiver coil since they can be detected in the presence of the Larmor RF frequency ω_0 and the phase modulating low frequency Ω by rejecting these latter frequencies with circuit filters and/or by detection, heterodyning, and homodyne demodulation techniques employed in standard radio receivers. This received first sideband voltage is maximized if the argument of the first sideband is adjusted so that:

$$J_1\left(\frac{\gamma' h_p}{\Omega}\right) \cong J_1(1.8) \cong 0.582(\text{Abramowitz, op. cit. p. 390})$$

yielding

$$\Omega = \frac{\gamma\prime h_p}{1.8} = \frac{\gamma\mu_0 h_p}{1.8} = \frac{\gamma b_p}{1.8} = \frac{2\pi(42.589x10^6)}{1.8} \cdot \; b_p \;,$$

then the side band frequency $f_p \cong 23.6x10^6 \cdot b_p$ where $2\pi f_p = \Omega$. The peak excursion of the magnetic moment (spin) from the plane containing $H_0$and orthogonal to h_n is ± 1.8 radians, or ± 103 degrees.

[0034] Thus, Ω and h_p are so defined but are independent of the main magnetic field strength H_0 or Larmor frequency $\gamma' H_0 = \gamma\mu_0 H = \gamma B_0 = \omega_0$.

[0035] Flow Meter Application

[0036] If the magnetic moments (spins) $\vec{\mu}$ dwell in a space containing H_0 and h_p for a time sufficient to create significant magnetic field intensity (Slichter, op. cit. Ch.2.11, p.51) and then move through a space additionally containing h_n for a dwell time such that nutation occurs through an angle $\theta = \pi$, maximum energy is absorbed by the magnetic moments (spins) $\vec{\mu}$ (Balanis, op. cit. p.86, eq.2-81).

[0037] A medium (lattice) containing a distribution of magnetic moments $\vec{\mu}$ (spins) traversing a conduit of length L_1, in which these magnetic moments are subjected to both a strong magnetic field intensity $\overrightarrow{H_0}$ and co-aligned weak component h_p, sinusoidally varying with a frequency Ω, will absorb energy from the magnetic moments by first order kinetics, opposed by random thermal motion, creating a magnetic field intensity $\vec{m}$ such that

$$\widehat{\vec{m}}=\widehat{\vec{\chi}}\left(\vec{H}_0 + \vec{h}_p \cos\Omega t\right)\left(1 - e^{-t/T_1}\right),$$

(where $\widehat{\ }$ denotes both spatial vector and temporal phasor) (Slichter, op. cit., p.51-59, ch.2.11), $\widehat{\vec{\chi}}$ being the complex susceptibility (Slichter, op. cit., p.35-39, ch.2.8), T_1 the spin-lattice relaxation (Slichter, op. cit., p.8, eq.1.31)

[0038] The sensitive volume V is the space occupied by the magnetic fields $H_0, h_p,$ and $h_n,$ as previously defined. If the spins occupy the volume V for the mean time τ the flow rate is V/τ. After the mean time τ the spins are at colatitude ("flip" angle) θ and phase angle φ as governed by $\gamma' h_n \tau = F(\theta\backslash\varphi)$. Then

$$\frac{V}{\tau} = \gamma' \frac{h_n}{F(\theta \backslash \varphi)} V$$

the detectable transverse magnetization is $M = \int_0^\theta m \sin\theta \, d\theta \quad M = m(1 - \cos\theta)$. For maximum M, $\frac{dM}{d\theta} = m \sin\theta = 0; \theta = n\pi. \; F(\pi \backslash \varphi) = 2F(\pi/_2 \backslash \varphi) = 2K$ (Abramowitch, loc. cit., 17.4.3) K is a complete elliptic integral of the first kind. φ is the modular angle (Abramowitch, loc. cit., 17.2.1). φ is periodic, $\varphi = \frac{\gamma h_p}{\Omega} \sin \Omega t$, a formulation not treated elsewhere.

$$K \triangleq \int_0^{\pi/2} [1 - (sin^2\varphi)(sin^2\theta)]^{-1/2} d\theta \text{ (Abramowitch, loc. cit., 17.3.1)}$$

$$\text{if } \frac{d\theta}{dt} \ll \Omega, \text{then } \frac{d\theta}{d\varphi} \cong o. \; sin^2\varphi = 1 - cos^2\varphi \triangleq m.$$

$$\cos\varphi \overset{m}{=} \frac{1}{2\pi} \int_{-\pi}^{+\pi} \cos\left(\frac{\gamma h_p}{\Omega} \sin \Omega t\right) d(\Omega t) = \frac{1}{\pi} \int_0^\pi \cos\left(\frac{\gamma h_p}{\Omega} \sin \Omega t\right) d(\Omega t)$$

But $J_n(z) = \frac{1}{\pi} \int_0^\pi \cos(z \sin\varphi - n\varphi) d\varphi$; (Bessel) (Jahnke and Emde, op. cit., p150)

$$J_0\left(\frac{\gamma h_p}{\Omega}\right) = \frac{1}{\pi} \int_0^\pi \cos\left(\frac{\gamma h_p}{\Omega} \sin \Omega t\right) d(\Omega t)$$

$$m = 1 - J_0^2(1.8) = 1 - (.340)^2 = .884 \; ; \; K = \int_0^{\pi/2} [1 - .884 sin^2\theta]^{-1/2} d\theta = 2.5$$

$$V/_\tau = \left[\frac{\gamma'}{2(2.5)}\right] V h_n = \left[\frac{\gamma}{2(2.5)}\right] V b_n = \left[\frac{2\pi(42.6x10^6)}{2(2.5)}\right] V b_n \text{ Tesla}$$

$$V/_\tau = 53.5x10^6 V b_n \text{ Tesla} = 5.35x10^3 V b_n \text{ Gauss}$$

[0039] Noise Budget

1) Major Components:

   a) Main Magnet plus RF shielding

      i) Resistive — D.C. power supply and hot coil can be source of RF noise mutually coupled to receiver coil.

      ii) Permanent — Thermal noise capacitively coupled to receiver coil.

      iii) Superconducting — minimal noise.

iv) Hybrid — as above.

b) Phase Modulating Coil and ELF Power Supply

   i) Stable C.W. ELF (1-10 kHz) Power Supply must be monochromatic and with adjustable but constant current output.

   ii) Phase Modulating Coil of low resistance to reduce thermal noise.

c) RF Transmitter Coil and Adjustable RF Power Supply

   i) RF Transmitter Coil of low resistance to reduce thermal noise.

   ii) Monochromatic RF Power Supply auto-tuned to the slightly variable Larmor frequency.

d) RF Receiver Coil and RF Receiver Circuit.

   i) RF Receiver Coil of low resistance forming a very high Q resonant circuit.

   ii) RF Receiver Circuit impedance matched to RF Receiver coil with cross-correlation to RF Power Supply frequency to extract received signal from received noise.

e) Controller Circuit controls RF Transmitter Power Supply Larmor frequency and measured value of current output to RF Transmitter Coil to achieve maximum RF Receiver Circuit output.

2) Schedule of Major Components:

   a) Main Magnet $L_{2,}$ Magnetizing Magnet L_1
   b) Phase Modulating Coil
   c) Phase Modulating Coil Power Supply, ELF (1-10 kHz)
   d) RF Transmitter Coil
   e) RF Transmitter Coil Power Supply, Larmor frequency
   f) RF Receiver Coil
   g) RF Receiver Circuit
   h) Controller Circuit

i) Conduit and Mechanical Supports and Seals

3) Noise Contributions in Order of Severity:

   a) The two power supplies should be monochromatic dedicated circuitry.

   b) The three coils must be high Q, low resistance, low noise.

   c) The receiver coil must be a narrow passband resonant circuit that does not oscillate when impedance matched to the receiver.

[0040] Signal Strength

[0041] Signal strength varies as the square root of the Main Magnet L_2 field strength, and linearly with the area of the Receiver Coil, the latter being a function of conduit diameter. Field homogeneity in the receiver section L_2 is a function of the shape of the Main Magnet. The dwell time τ_1 in the magnetizing section of length L_1 must be a significant fraction of the spin-lattice relaxation time of the moving material T_1.

[0042] Rangeability

[0043] The Range of Velocity v should be converted to dwell times τ_1 and τ_2 for lengths L_1 and L_2, for each flow range application, and compared τ_1 to T_1 spin-lattice and τ_2 to T_2^* spin-spin relaxation times for each class of material measured.

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